An Introduction to S-Parameters

APPLICATION BRIEF
LAB-1025

March 11, 2009

Summary

This application brief is intended to help oscilloscope users develop a basic understanding of S-parameters. This information will be useful in operating LeCroy's Eye Doctor™ II Advanced Signal Integrity Tools.

Our starting point is a simple two port network shown in Figure 1. The device under test (DUT) is a "black box" network, it can have resistors, capacitors, inductors, transistors, transmission lines, etc. Measurements can be made at each "port" to characterize the network, which can be single-ended or differential. The load impedance, Z_L, and the source impedance, $Z_{S,}$ are typically some characteristic impedance, most typically this is 50 Ω and we will use that value for our discussion.

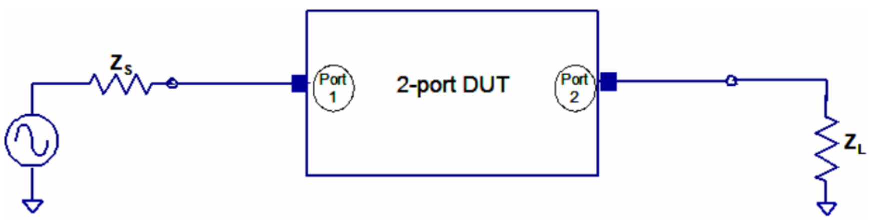


Figure 1: *A simple 2-port device under test (DUT), ZL is the load impedance and Zs is the source impendence of generator*

Consider what happens when power is applied from a sine source. This is shown in Figure 2. Incident Power is applied from the generator (a1) some power is reflected at the input of the DUT (b1). Reflections are caused by impedance mismatch between the source impedance and impedance looking into Port 1. Some power is transmitted at the output of the DUT (b2).

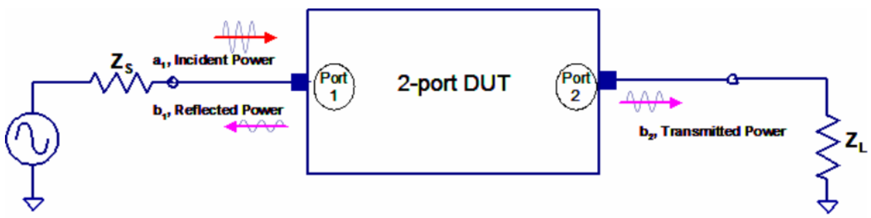


Figure 2: *Simplified Microwave Power Transfer in the two port network*

‘Scattering’ or ‘s’ parameters are a measure of reflected power and transmitted power in a network as a function of frequency. The “Network” could be a coax cable, passive antenna, active amplifier, microwave filter, etc. S-parameters have magnitude and phase Typically, magnitude is measured in dB, phase is measured in degrees.

The naming scheme for S-parameters is of the form:
S <output port> <input port>

“Forward” S-parameters can be measured by driving the input and terminating the output:

S_{11}= b_1/a_1=Reflected Power/Incident power

S_{11} is the input reflection coefficient with the output port terminated by a matched load.

Figure 3 shows examples of typical forward s parameter plots. **S_{11}** is the top trace.

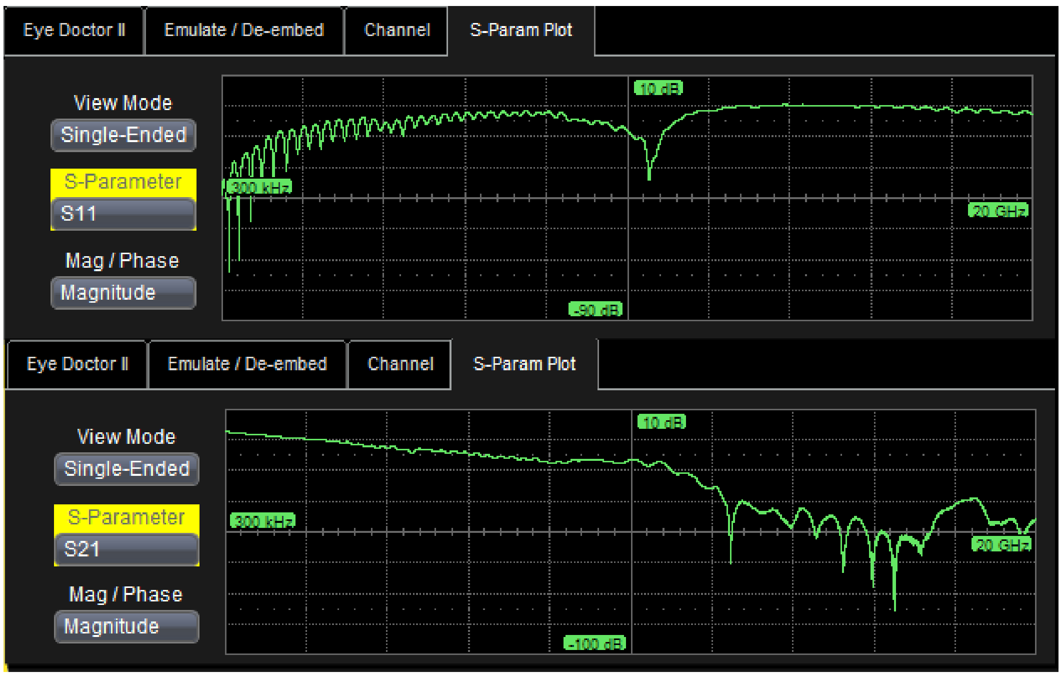


Figure 3: *Plots of the forward S-parameters S_{11} and S_{21}*

Example S21 = transmission from port 1 to port 2

S-parameters are typically measured using a Vector Network Analyzer (VNA). The VNA measures S-parameters over frequency by sweeping the frequency of the input, it has the ability to separate transmitted and reflected power using directional couplers for power measurements. An alternative measurement is to use a time domain reflectometer (TDR) , such as the the ST-20 TDR module in the LeCroy WaveExpert Sampling oscilloscope.

S_{21}= b_2/a_1=Transmitted/Incident power

$S_{21}$is the forward transmission gain with the output port terminated by a matched load.

In Figure 3, **S_{21}**is the bottom trace.
These plots are displayed on the scope by selecting the “View Response” button in the Eye Doctor™ II “Emulate” dialog box. This display will show the user selected S-parameter.

"Reverse" S-parameters are measured by driving the output and terminating the input as shown in Figure 4.

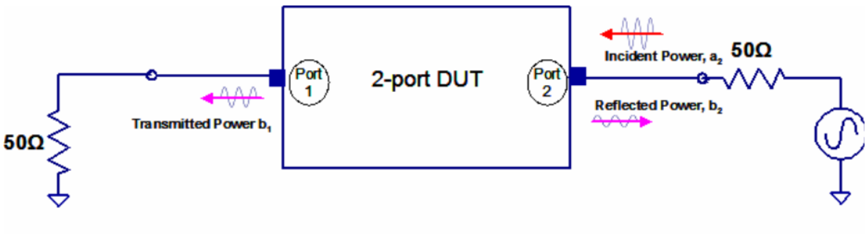


Figure 4: The measurement setup for determining the "reverse" S-parameters

S_{22}= b_2/a_2 =Reflected Power/Incident power.

S_{22} is the output reflection coefficient with the input port terminated by a matched load.

S_{12}= b_1/a_2=Transmitted/Incident power.

S_{12} is the reverse transmission gain with the input port terminated by a matched load.

Figure 5 shows a typical reverse s parameter plot S_{22} is the top trace and S_{12} is bottomtrace.

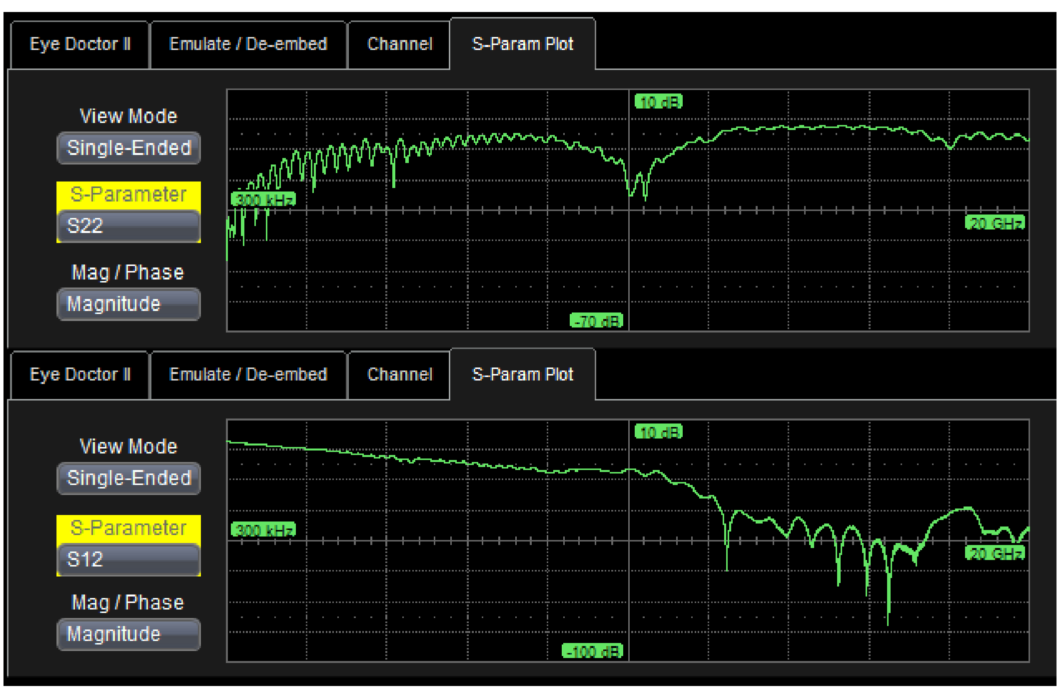


Figure 5: Plots of the reverse S-parameters S_{22} and S_{12}.

We can now show the relationships which totally describe the 2 port device in terms of the inputs, outputs and S-parameters.

$$b_1=S_{11}a_1+S_{12}a_2$$
$$b_2=S_{21}a_1+S_{22}a_2$$

These relationships along with a graphic description of the inputs and outputs are shown in Figure 6.

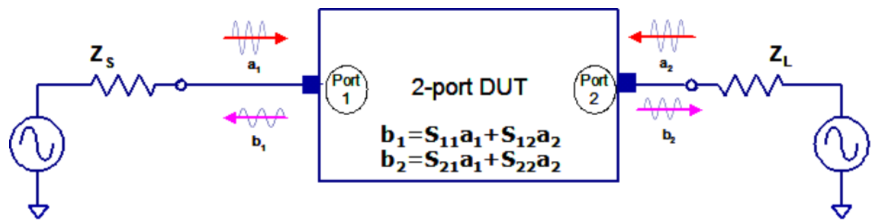


Figure 6: The general two port s parameter relationships

Before moving forward with the discussion of S-parameters we are going to review differential measurements. Consider a differential amplifier illustrated in Figure 7.

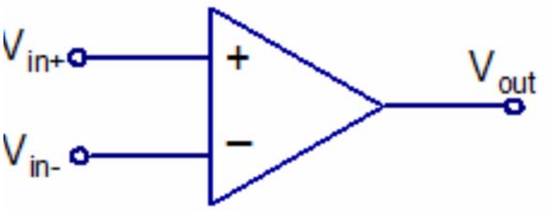


***Figure 7*: A differential amplifier**

In a differential system, like this amplifier, common mode signals are in phase
And differential mode signals are 180° out of phase. The input ports are treated as differential inputs and outputs (+ and – terminals). For a differential amplifier:

$$\mathbf{V_{out} = A_D(V_{in+} - V_{in-}) + A_{CM}(V_{in+} + V_{in-})/2}$$

A_D *is the differential mode gain; i.e. the gain for a differential mode signal*
A_{CM} *is the common mode gain, i.e. the gain for a common mode signal*

Common Mode Rejection Ratio
CMRR = $\mathbf{A_{CM}}$ / $\mathbf{A_D}$
CMRR measures how much of a common-mode input get converted to the output, this should be small for a differential amplifier.

Differential transmission is now commonly used for most high speed serial data communications links. Because of that we will extend our investigation of S-parameters into four port networks which can be viewed as 2 port differential networks. We'll start by looking at a four port single ended network shown in Figure 8.

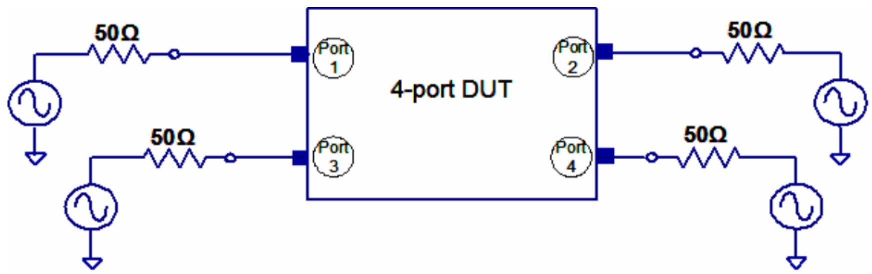


Figure 8: *A general 4 port Measurement Setup*

Single ended s-parameter measurements for a four port network are straightforward—just like two port case, only including more S-parameters. In matrix format the S-parameters for a four port network will look like this:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{21} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{24} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Differential measurements are more complex for these measurements we need "Mixed-Mode" S-parameters. A measurement setup for mixed mode S-parameters is shown in Figure 9. These measurements require some care because coupling between differential lines can influence measurements. Also, mode conversion influences how much common mode gets converted to differential mode and vice versa.

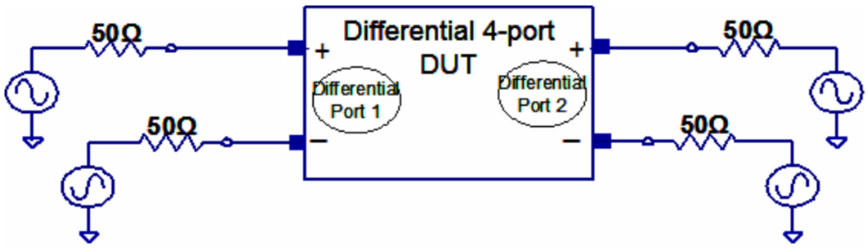


Figure 9: *A measurement setup for mixed mode S-parameter measurements*

By taking four port single ended measurements, Mixed Mode S-parameters can be calculated using a matrix transformation. This is a simple but it is outside the scope of this paper and will not be covered here. References are available.

The output of the conversion process looks like this:

$$\left[\begin{array}{cc|cc} S_{dd11} & S_{dd12} & S_{dc11} & S_{dc12} \\ S_{dd21} & S_{dd22} & S_{dc21} & S_{dc22} \\ \hline S_{cd11} & S_{cd12} & S_{cc11} & S_{cc12} \\ S_{cd21} & S_{cd22} & S_{cc21} & S_{cc22} \end{array}\right]$$

In general there are four types of mixed mode S-parameters:

Differential Mode to Differential Mode S_{dd})
Differential Mode to Common Mode (S_{dc})
Common Mode to Differential Mode (S_{cd})
Common Mode to Common Mode (S_{cc})

For example S_{dd21} is the differential mode gain from differential port 1 to differential port 2

We can now show the relationship between the inputs and outputs of a two port differential measurement as:

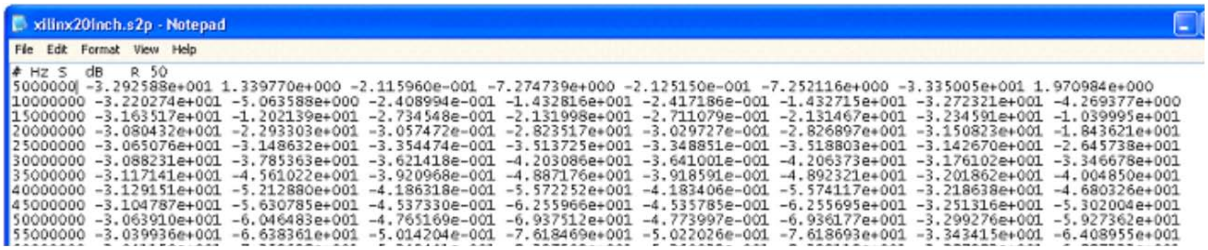
$$\begin{bmatrix} b_{d1} \\ b_{d2} \\ b_{c1} \\ b_{c2} \end{bmatrix} = \begin{bmatrix} S_{dd11} & S_{dd12} & S_{dc11} & S_{dc12} \\ S_{dd21} & S_{dd22} & S_{dc21} & S_{dc22} \\ S_{cd11} & S_{cd12} & S_{cc11} & S_{cc12} \\ S_{cd21} & S_{cd22} & S_{cc21} & S_{cc22} \end{bmatrix} \begin{bmatrix} a_{d1} \\ a_{d2} \\ a_{c1} \\ a_{c2} \end{bmatrix}$$

Where the inputs a_{d1} is the differential input to a1 and a_{c1} is the common mode input to a1. The output terms are similarly annotated.

You have learned that S-parameters are used to describe a network. In LeCroy's Eye Doctor II S-parameters are used to describe fixtures, backplanes, and other circuit elements. Eye Doctor II uses these descriptions to remove selected circuit elements from the measurement or to emulate them. Characterizing circuit elements with S-parameters can be done in several ways.

S-parameters can be measured directly using a Vector Network Analyzers(VNA) or using a LeCroy WaveExpert sampling oscilloscope with time domain reflectometry (TDR) and time domain transmission (TDT) options. Software like computer aided design (CAD) Tools can be used to calculate S-parameters examples are HFSS Field Solve Tools or the ADS Simulator.

All these tools create Touchstone files that are used by Eye Doctor II. The Touchstone standard is a file format used for s parameter descriptions of circuits and devices. It uses file extension of the form: .sNp. Where N is number of ports measured. A 2-port would be .s2p, or a 4-port .s4p, etc. Figure 10 shows a typical Touchstone file.



```
# Hz S   dB    R 50
5000000 -3.292588e+001 1.339770e+000 -2.115960e-001 -7.274739e+000 -2.125150e-001 -7.252116e+000 -3.335005e+001 1.970984e+000
10000000 -3.220274e+001 -5.063588e+000 -2.408994e-001 -1.432816e+001 -2.417186e-001 -1.432715e+001 -3.272321e+001 -4.269377e+000
15000000 -3.163517e+001 -1.202139e+001 -2.734548e-001 -2.131998e+001 -2.711079e-001 -2.131467e+001 -3.234591e+001 -1.039995e+001
20000000 -3.080432e+001 -2.293303e+001 -3.057472e-001 -2.823517e+001 -3.029727e-001 -2.826897e+001 -3.150823e+001 -1.843621e+001
25000000 -3.065076e+001 -3.148632e+001 -3.354474e-001 -3.513725e+001 -3.348851e-001 -3.518803e+001 -3.142670e+001 -2.645738e+001
30000000 -3.088231e+001 -3.785363e+001 -3.621418e-001 -4.203086e+001 -3.641001e-001 -4.206373e+001 -3.176102e+001 -3.346678e+001
35000000 -3.117141e+001 -4.561022e+001 -3.920968e-001 -4.887176e+001 -3.918591e-001 -4.892321e+001 -3.201862e+001 -4.004850e+001
40000000 -3.129151e+001 -5.212880e+001 -4.186318e-001 -5.572252e+001 -4.183406e-001 -5.574117e+001 -3.218638e+001 -4.680326e+001
45000000 -3.104787e+001 -5.630785e+001 -4.537330e-001 -6.255966e+001 -4.535785e-001 -6.255695e+001 -3.251316e+001 -5.302004e+001
50000000 -3.063910e+001 -6.046483e+001 -4.765169e-001 -6.937512e+001 -4.773997e-001 -6.936177e+001 -3.299276e+001 -5.927362e+001
55000000 -3.039936e+001 -6.638361e+001 -5.014204e-001 -7.618469e+001 -5.022026e-001 -7.618693e+001 -3.343415e+001 -6.408955e+001
```

Figure 20: *A sample of a two port Touchstone file*

Each line of a 2-port data set has the following structure:
<frequency value> <Mag S11> <Phase S11> <Mag S21> <Phase S21> <Mag S12> <Phase S12> <Mag S22> <Phase S22>

Eye Doctor II uses these files in the Emulate / De-embed function shown in Figure 11. The Touchstone file is referenced through the S-Parameter Filename field in the Channel and De-embed dialog boxes. Pressing the View Response button in the Channel dialog displays a user displayed s parameter as a function of frequency.

As you have observed, S-parameters are an efficient way to describe circuit behavior and they are used quite effectively in LeCroy's EyeDoctor II Advanced Signal Integrity Tool.

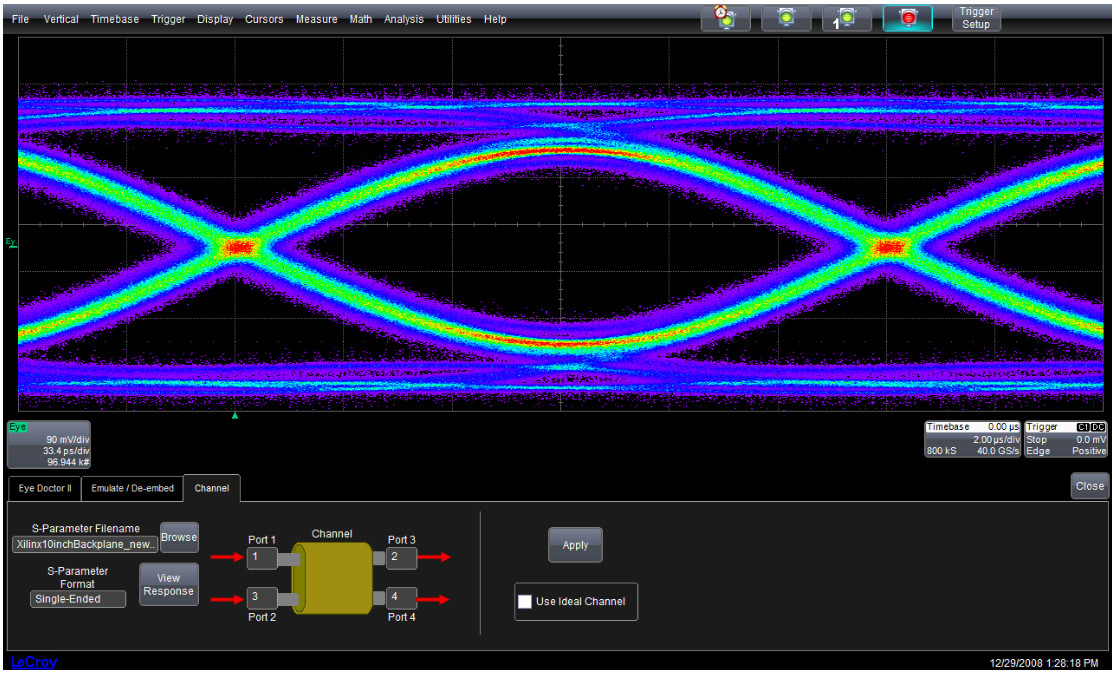


Figure 11: *The emulate/de-embed dialog box for EyeDoctor II showing the link to the Touchstone file describing the S-parameters of the signal channel.*